Question Bank For PG Course

Mathematics
Paper-9A(i)(NEW)
Special Paper: Pure Mathematics
ADVANCED COMPLEX ANALYSIS: PGMT-IXA

Question 1

If $f(z) = u + iv$ is analytic in a region R, then find the value of $\frac{\partial(u,v)}{\partial(x,y)}$.

Question 2

Given that $f(z)$ is analytic at $z = \alpha$ and $f'(\alpha) = f''(\alpha) = f'''(\alpha) = 0$ but $f^{(iv)}(\alpha) \neq 0$. Then find the magnifies angles of $f(z)$ at $= \alpha$.

Question 3

What is the sufficient condition for convexity of a function (x) ?

Question 4

Find the order of the function $f(z) = e^{z^n}$ (n is a positive integer)

Question 5

Find the exponent of convergence of the zeros of $f(z) = \cos z$.

Question 6

Given that $f(z)$ is analytic in $|z| < 1$, with a zero of order n at the origin. Also $|f(z)| \leq 1$ for all z in $|z| < 1$. Then which of the following inequalities holds
      1. $|f(z)| \leq |z|^n, |z| > 1$
2. $|f(z)| \geq |z|^n, |z| < 1$
3. $|f(z)| \geq |z|^n, |z| > 1$
4. $|f(z)| \leq |z|^n, |z| < 1$

Question 7

Find the canonical product of the function $f(z) = \sin z$.

Question 8

What is Legendre's duplication formula?

Question 9

Given that $f(z)$ is an entire function which never vanishes. Then there exists an entire function $g(z)$ such that

1. $f(z) = g(z)$ for all z
2. $f(z) = \dfrac{1}{g(z)}$ for all z
3. $f(z) = e^{g(z)}$ for all z
4. $f(z) = e^{-g(z)}$ for all z.

Question 10

An entire function $f(z)$ is said to be of finite order if for some k and some $R > 0$ and for all z with $|z| \geq R$

1. $|f(z)| \geq \exp\left(|z|^k\right)$
2. $|f(z)| \leq \exp\left(|z|^k\right)$
3. $|f(z)| \geq \exp\left(|z|^{2k}\right)$
4. $|f(z)| \leq \exp\left(|z|^{2k}\right)$

Question 11

Find the branch points of the function
$$f(z) = (z^3 - z)^{1/3}$$

Question 12

Which of the following function is single valued?

1. $\left(z^{\frac{1}{2}}\right)^3$
2. $(z^3)^{\frac{1}{2}}$
3. $(z^2)^{\frac{1}{2}}$
4. $(z^{1/2})^2$

Question 13

Given that $f(z)$ is a nonconstant analytic function in a domain D. Then examine the harmonocity of $|f(z)|$.

Question 14

Find the the harmonic conjugate of the function $u(x, y) = \log\sqrt{x^2 + y^2}$

Question 15

Find the branch points of the function
$f(z) = \sin z^{1/2}$

Question 16
If $f(z) = u + iv = z^2$ then what is the
value of $\frac{\partial(u,v)}{\partial(x,y)}$?

Question 17
Find the value of the infinite product
$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots$$

Question 18
Determine the map which transforms
the vertical semi-infinite strip
$-\frac{\pi}{2} < u < \frac{\pi}{2}, v > 0$ onto the upper half
plane $Im\, z > 0$.

Question 19
What is the order of the function
$f(z) = \cos\sqrt{z}$?

Question 20
Find the exponent of convergence of
the zeros of $f(z) = \sin z$.

Question 21
Find the residue of the Gamma function
at $-n$.

Question 22
Find the critical points of the mapping
$f(z) = e^{2z} - 2iz + 3.$

Question 23
What is the value of $\Gamma(z)\Gamma(1-z)$?

Question 24
What is the order of the function
$f(z) = e^{z^2}$?

Question 25

Which of the following function is
single valued?

Question 26

Let $f(z)$ and $g(z)$ are entire functions
having order 2 and 3 respectively.
Then what will be the maximum order
of $f(z) - g(z)$?

Question 27

Find the harmonic conjugate of the
function $u(x, y) = 2x - x^3 + 3xy^2$.

Question 28

Find the branch points of the function
$f(z) = z^{1/2}\cos z^{1/2}$.

Question 29

Find the branch points of the function
$f(z) = (z^3 + z^2 - 6z)^{1/2}$.

Question 30

What is the Legendre's duplication
formula ?