Question Bank For PG Course

Mathematics
Paper-9B(i)(OLD)
Special Paper: Pure Mathematics
TOPOLOGICAL GROUP : PGMT-IXB(OLD)

Question 1

Let P be an open set in topological group G and A be any subset of G. Then which of the followings is/are true?

(i) uP, Pu are open set in G for any member $u \in G$

(ii) AP and PA are open sets in G

(iii) $P - A$ is a closed set in G

Question 2

Which of the followings is/are true?

(i) The circle group is homeomorphic with the unite circle in $\mathbb{C}$.

(ii) If G be a locally compact Hausdorff abelian topological group, then its dual group G^* is also a similar topological group

(iii) Let G be a topological group then G^* be a topological group.

Question 3

Let F be a closed set and C be a compact set in a topological group. Which of the followings is/are true?

(i) FC is a closed set

(ii) CF is a closed set

(iii) CF or FC may not closed.

Question 4

Fill in the blank.

A topological group that is Hausdorff is .....

(i) completely regular

(ii) regular

(iii) T_1

Question 5

Which of the followings is/are true?

(i) Let H be a sub-group of a topological group G, then its closure $\overline{H}$ is so.

(ii) A sub-group$H$ of a topological group G is open iff its interior $int(H) \neq \emptyset$.

(iii) A sub-group$H$ of a topological group G is open iff its interior $int(H) = \emptyset$.

Question 6

Fill in the blank.

The centre of a Hausdorff topological group is a ……. Normal sub-group.

(i) open

(ii) closed

(iii) may not be closed

Question 7

Which of the followings is/are true?

(i) $M_n(\mathbb{R})$ is Hausdorff topological group that is compact

(ii) $M_n(\mathbb{R})$ is Hausdorff topological group that is not compact

(iii) $M_n(\mathbb{R})$ is Hausdorff topological group that is locally compact

(iv) $M_n(\mathbb{R})$ is Hausdorff topological group that is not locally compact

Question 8

Which of the followings is/are true?

(i) $G_n(\mathbb{R})$ is an open set in topological group $M_n(\mathbb{R})$.

(ii) $G_n(\mathbb{R})$ is an closed set in topological group $M_n(\mathbb{R})$.

(iii) $O_n(\mathbb{R})$ is a compact topological sub group of $G_n(\mathbb{R})$.

Question 9

Let $\{x_n\}$ and $\{y_n\}$ be two Cauchy sequences in a Banach Algebra X. Is the sequence $\{x_n y_n\}$ Cauchy?

Question 10

Let f be a complex homomorphism on a Banach Algebra X and $x \in X$ with $||x|| \neq 1$. Then find $f(x)$.

Question 11

Let X be a Banach Algebra with identity. Is the set of all invertible elements of X open or closed?

Question 12

Is multiplication operator in a Banach Algebra continuous?

Question 13

Let M be a maximal ideal in a commutative Banach Algebra X with identity$e$. Find the value of $e(M)$.

Question 14

Let M be a maximal ideal in a commutative Banach Algebra X with identity. Is X/M a Banach Algebra?

Question 15

Which of the following statement is always true about an ideal I of the commutative Banach Algebra X

1. For any $x \in X$, $x\ belongs\ to\ I$
2. Identity e of X belongs to I
3. For any $x \in X\ and\ y \in I, then\ xy \in I$

Question 16

Fill in the blank:
Let G be a topological group. Then the map $G \to G$ given by $x \to x^{-1}$ are.....

(i) Continuous
(ii) Self homeomorphism
(iii) Differentiable
(iv) Smooth

Question 17

Which of the following is/are not topological group?

(i) Set of all $n \times n$ real matrices with addition operation

(ii) Set of all $n \times n$ complex matrices with addition operation

(iii) Set of all reals R with upper limit topology.

Question 18

Let P be an open set in a topological group G. Which of the following is/are true?

(i) $(G - P)u$ is closed for $u \in G$

(ii) uP is closed for $u \in G$

(iii) uP is open for $u \in G$

Question 19

Let E and W be two compact subsets of a topological group G. Which of the following is/are true?

(i) EW is also compact

(ii) EW is not always compact

(iii) EW is open

Question 20

Let (X, T) be a topological space. Which of the followings is/are true?

(i) If every singleton set is closed then (X, T) is T_1

(ii) If every net in X converges to at most one point in X, then (X, T) is T_2

(iii) If (X, T) is T_2, then every sequence is convergent.

Question 21

Let G be a T_1 topological group. Which of the following is/are true?

(i) Every net in G converges to at most one point in G.
(ii) G is T_0 space
(iii) Every singleton set in G is closed.

Question 22

Let H be a subgroup of a topological group G. Which of the following is/are true?

(i) If H is normal in G, then $\overline{H}$ is so.
(ii) If H is open, then H is also closed in G.
(iii) $\overline{H}$ is closed in G.

Question 23

Fill in the blank. The centre of a Hausdorff topological group is.....

(i) Open
(ii) Normal and open
(iii) Normal and closed

Question 24

Fill in the blank.
Let G be a topological group and H a sub-group of G. Then G/H is a discrete space if and only if H is.....

(i) Closed
(ii) Open
(iii) Neither open nor closed
(iv) Normal subgroup

Question 25

Fill in the blank.
A topological group is locally compact topological group iff its identity has.....

(i) Compact neighbourhood
(ii) Open neighbourhood
(iii) Closed neighbourhood

Question 26

Let $f: G \to H$ be a continuous homomorphism between two topological groups. Which of the following is/are not true?

(i) f is an open mapping
(ii) f is a homeomorphism
(iii) f is a closed mapping

Question 27

Let $f: G \to H$ be a homomorphism between two topological groups. Which of the following is/are true?

(i) If A is symmetric in G, then $f(A)$ is symmetric in H
(ii) For $A \text{ and } B \subset G$, $\overline{f(A)f(B)} \subset \overline{f(AB)}$
(iii) For $A \text{ and } B \subset G$, $\overline{f(AB)} \subset \overline{f(A)f(B)}$

Question 28

Is the set of all invertible elements of a Banach Algebra X open or closed?

Question 29

Let Z denotes the set of all topological divisors of zero in Banach Algebra X and G is the set of all invertible elements of X. Which of the following is/are true?

(i) X\G = Z
(ii) $Z \subset$X\G
(iii) X\G $\subset Z$

Question 30

Find the value of

$$\int_0^\infty \frac{(\cos \lambda x)}{\lambda^2 + 1} d\lambda$$