QP Code : 24/PT/11/XVIIB

POST-GRADUATE DEGREE PROGRAMME
Term End Examination — December, 2024
ECONOMICS
Paper-XVIIB : ADVANCED STATISTICS

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given accuracy and relevance in the answer. Marks will be deducted for spelling, untidy work and illegible handwriting. The weightage for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

1. Answer any *four* of the following questions : $2\frac{1}{2} \times 4 = 10$

   a) If $\hat{\theta}$ is an unbiased estimator of θ then :

      (i) Does $\hat{\theta}^2$ be unbiased estimator of θ^2 ?

      (ii) Does $\sqrt{\hat{\theta}}$ be unbiased estimator for $\sqrt{\theta}$?

   b) What is Type-I error ? How is it different from Type-II error ?

   c) Arithmetic Mean and Standard Deviation of a binomial distribution are respectively 4 and $\sqrt{\frac{8}{3}}$. Find the values of *n* and *p*.

   d) What is the 'Central Limit Theorem' ?

   e) Distinguish between random and non-random sample.

   f) An Urn contains six red and four white balls. Two balls are drawn without replacement. What is the probability that the second ball is red, if it is known that the first is red ?

2. Answer any *four* of the following questions : $5 \times 4 = 20$

   a) An article manufactured by a company consists of two Parts : I and II. In the process of manufacture of Part I, 9 out of 100 are likely to be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of Part-II. Calculate the probability that the assembled article will not be defective.

b) If x_1, x_2, x_3 be 3 random samples drawn from $N(\mu, \sigma^2)$, and $T_1 = x_1 + x_2 - x_3$ and $T_2 = 3x_1 - 4x_2 + 2x_3$, which estimator would you choose for μ ?

c) X is a discrete random variate having probability mass function :

X	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

i) Determine the constant k ;

ii) Find $P(X < 6)$;

d) Find the mean and variance of Poisson distribution.

e) A random sample of size 10 was drawn from a normal population with an unknown mean and a variance of 44·1 $(\text{inch})^2$. If the observations are (in inches) : 65, 71, 80, 76, 78, 82, 68, 72, 65 and 81, obtain 95% Confidence Interval for the population mean.

f) A random sample of size 20 from a normal population gives a sample mean of 42 and sample standard deviation of 6. Test the hypothesis that the population mean is 44. State clearly the alternative hypothesis you allow for and the level of significance adopted.

[Note : the table value of $t_{0\cdot025} = 2\cdot09$ and $t_{0\cdot005} = 2\cdot86$ for 19 degrees of freedom]

3. Answer any *two* of the following questions : $10 \times 2 = 20$

a) i) Three identical boxes I, II, III contain respectively 4 white and 3 red balls, 3 white and 7 red balls, and 2 white and 3 red balls. A box is chosen at random and a ball is drawn out of it. If the ball is found to be white, what is the probability that box II was selected ?

ii) A box contains 4 white and 6 black balls. If 3 balls are drawn at random, find the mathematical expectation of the number of white balls. 5 + 5

b) i) Given that $E(T_1) = \theta_1 + \theta_2$ and $E(T_2) = \theta_1 - \theta_2$. Find the unbiased estimates for θ_1 and θ_2.

ii) Let $x_1, x_2 \ldots, x_n$ be n random samples drawn from normal population $N(\mu, \sigma^2)$. Show that sample mean is unbiased for population mean but sample variance is not unbiased for σ^2, where sample variance $S^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$. 3 + 7

c) i) On the basis of a random sample, find the maximum likelihood estimator of the parameter of a Poisson distribution.

ii) A random sample of size 10 was taken from a normal population, whose variance is known to be 7·056 sq.inches. If the observations are (in inches) 65, 71, 64, 71, 70, 69, 64, 63, 67 and 68, test the hypothesis that the population mean is 69 inches. Also obtain 99% confidence limit for the population mean.

(Note : $|z| \geq 1 \cdot 96$ at 5% level and $|z| \geq 2 \cdot 58$ at 1%) 5 + 5

d) Write short notes on (any *two*) : $5 \times 2 = 10$

i) Chi-square distribution and its properties

ii) Stratified random sampling

iii) Maximum Likelihood estimator

iv) Sampling bias.