POST-GRADUATE DEGREE PROGRAMME

Term End Examination — December, 2024

MATHEMATICS

Paper-2B : COMPLEX ANALYSIS

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

(*Symbols have their own meanings.*)

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Find the roots of $(1+z)^5 = (1-z)^5$.

   b) Show that $f(z) = z^2$ is uniformly continuous in the region $|z| < 1$, but not uniformly continuous on $\mathbb{C}$.

   c) Show that the function $f(z) = \bar{z}$ is continuous everywhere but non-analytic everywhere.

   d) Evaluate : $\int_\gamma \bar{z}\, dz$, where γ is the upper half of the circle $|z| = 1$ from $z = -1$ to $z = 1$.

   e) Find the radius of convergence of the following power series : $\sum_{n \in N} (3+4i)^n z^n$.

   f) Given that α is a simple pole of *f*. Prove that $Res(f; \alpha) = \lim_{z \to \alpha} (z - \alpha) f(z)$.

   g) Find the fixed points of the Möbius transformation : $T(z) = \dfrac{2z-5}{z+4}$.

2. a) Establish the relation

   $$\frac{n}{2^{n-1}} = \prod_{k=1}^{n-1} \sin\left(\frac{k\pi}{n}\right), \; n \geq 2.$$

   b) Prove that the function $arg : \mathbb{C} \setminus \{0\} \to (-\pi, \pi]$ is not continuous function for all negative real numbers. 6 + 4

3. a) State and prove the necessary condition for differentiability.

   b) State and prove the M-L formula. 5 + 5

4. a) Let $p(z)$ be a non-constant polynomial and α be any complex number. Show that the equation $p(z)=\alpha$ has a solution in ₵.

   b) For what values of z, does the series $\sum_{n=1}^{\infty} \frac{1}{(z^2+1)^n}$ converges ? Moreover, find its sum. 5 + (3 + 2)

5. a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for $1 < |z| < 3$.

   b) State and prove the Cauchy residue theorem. 5 + 5

6. a) Evaluate : $\oint_{|z|=2} \frac{e^z dz}{z^4 - 1}$.

   b) State and prove the Rauche's theorem. 4 + 6

7. a) Evaluate : $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$.

   (using the idea of contour integration)

   b) Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2+y^2-4x=0$ onto the line $4x+3=0$. 6 + 4