

POST-GRADUATE DEGREE PROGRAMME
Term End Examination — December, 2024
MATHEMATICS

Paper-4A : NUMERICAL ANALYSIS

Time : 2 hours]

[Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. 1 and any *four* from the rest :

1. Answer any *five* questions : 2 × 5 = 10
 - a) Find the condition number of $f(x) = \frac{x}{1-x^2}$.
 - b) Define an ill-conditioned matrix. Give an example.
 - c) Define Chebyshev polynomial $T_n(x)$, $n \geq 0$ and write its recurrence relation.
 - d) Compute the total number of arithmetic operations (addition/subtraction and multiplication/division) in Gaussian algorithm for solving an $(n \times n)$ system of linear equations.
 - e) Define the degree of precision of a quadrature formula. Find the maximum possible value of the degree of precision for a quadrature formula with $(n + 1)$ nodes.
 - f) Show that the modified Euler's method is a second order scheme.
 - g) Let (λ, X) be an eigen of an $n \times n$ matrix A so that $AX = \lambda X$. What will be the corresponding eigen pair of the similar matrix $P^{-1}AP$?
2. Discuss the stability analysis of second order Runge-Kutta method and obtain the stability region of it. 10
3. a) Define Spline function. What are the advantages of Spline approximation over the ordinary interpolation. 2 + 3
 - b) Describe briefly 4-step Adam-Bashforth method for the numerical solution of the initial value problem $\frac{dy}{dx} = f(x, y)$, $y(a) = y_0$ in a finite interval $[a, b]$. 5

4. Fit a quadratic function in t correct to four significant figures by the method of least-squares for the following data :

t	0	0.5	1.0	1.5	2.0	3.0	5.0	8.0	10.0
x	3.85	2.95	2.63	2.33	2.24	2.05	1.82	1.80	1.75

10

5. Solve the following hyperbolic equation by finite difference method :

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0.$$

$u(x, 0) = 2x(1-x) \sin \pi x$, $u(x, 0) = 0$, $0 \leq x \leq 1$, $u(0, t) = u(1, t) = 0$, $t \geq 0$ with step size $h = k = 0.1$.

10

6. Describe the basic principle for Power Method for finding the dominant eigen pair of given square matrix $(A)_{n \times n}$. How the convergence rate of the method depends upon the magnitudes of its eigen values ? Can the method be extended to find out the second largest eigen pair ? How the method can be used for finding the least eigen pair of A ? State when the method fails.

5 + 2 + 2 + 1

7. a) Find cubic Spline function for the following data :

$$(-2, -8), (0, 0), (1, 1), (2, 8), \quad S''(-2) = -12, \quad S''(2) = 12. \quad 5$$

- b) Solve the following boundary value problem correct to three decimal places by a finite difference scheme :

$$y'' = xy' + 1; \quad y(0) = 1.5, \quad y(1) = 0.5, \quad h = 0.1 \quad 5$$