QP Code: 24/PT/13/VA

POST-GRADUATE DEGREE PROGRAMME

Term End Examination — December, 2024

MATHEMATICS

Paper-5A : PRINCIPLES OF MECHANICS

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

(*All Symbols have their usual meanings.*)

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : 2 × 5 = 10

   a) Define action of a mechanical system.

   b) What do you mean by Legendre's dual transformation ?

   c) Define generalized momentum corresponding to a generalized coordinate.

   d) Write down Hamilton's canonical equation in symmetrical form.

   e) State interrelation between Poisson bracket and Lagrange's bracket.

   f) State principle of linear and angular momentum.

   g) What do you mean by phase space ?

2. a) Show that the K . E. of a moving system can be expressed as

   $$T = \frac{1}{2}\sum_{i,\ j} a_{ij}\,\dot{q}^{\,i}\dot{q}^{\,j} + \sum_{i} b_i\,\dot{q}^{\,i} + c$$

   b) Derive Hamilton-Jacobi partial differential equation. 5 + 5

3. a) Find the deflection of a freely falling body from the vertical caused by earth's rotation.

   b) Show that the transformation

   $Q = \sqrt{q}\cos p, P = \sqrt{q}\sin p$

   represents a canonical transformation. Hence evaluate the new Hamiltonian of the system for which $T = \frac{1}{2}m\dot{q}^2$, $V = \frac{1}{2}kq^2$. 5 + 5

4. a) Derive Lagrange's equation of motion for a conservative and unconnected holonomic system.

   b) Show that generalized momentum is conserved corresponding to a cyclic coordinate. Write down Lagrange's equation of motion as the evolution of generalized momentum. 5 + 5

5. a) Solve the one dimensional harmonic oscillator

   $T = \frac{1}{2}\dot{q}^2, \; V = \frac{1}{2}\mu^2 q^2$

   using canonical transformation $(q, p) \to (Q, P)$ with the generating function $F_1 = \frac{1}{2}\mu q^2 \cot Q$.

   b) Show that $J_1 = \int_{S_2} \sum_i \mathrm{d}q_i \, \mathrm{d}p_i$, is invariant under canonical transformation where S_2 is a $2D$ surface in phase space. 5 + 5

6. a) Derive D'Alembert's principle from Hamilton's principle.

   b) The K.E. and P.E. of a particle are given by

   $T = \frac{1}{2}(\dot{x}^2 + \dot{y}^2)$ and

   $V = \frac{\alpha}{x^2} + \frac{\beta}{y^2} + \frac{\gamma}{r} + \frac{\gamma'}{r'} + \delta(x^2 + y^2)$, where α, β, γ, γ' and δ are constants and r, r' are distances of the particle from the points $(\mu, 0)$ $(-\mu, 0)$ respectively, μ being a constant.

   Show that the system can be converted into Liouville's type. 5 + 5

7. a) State and prove Brachistochrone problem.

   b) State and prove the principle of least action. 6 + 4