

POST-GRADUATE DEGREE PROGRAMME
Term End Examination — December, 2024
MATHEMATICS

**Paper-5B : ELEMENTS OF CONTINUUM MECHANICS & SPECIAL
THEORY OF RELATIVITY**

Time : 2 hours]

[Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. 1 and any *four* from the rest :

1. Answer any *five* questions : 2 × 5 = 10

- a) Two particles come towards each other, each with speed $0.9c$ with respect to the laboratory. What is their relative speed ?
- b) The state of stress at a point is given by

$$(T_{ij}) = \begin{pmatrix} T & aT & bT \\ aT & T & cT \\ bT & cT & T \end{pmatrix}$$

where a, b, c are constants and T is some stress value. Determine the constants a, b, c so that stress vector on a plane normal to

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \text{ vanishes.}$$

- c) The displacement in an elastic solid is given by $u_1 = a(X_1 + 2X_2 + 3X_3)$, $u_2 = a(-2X_1 + X_2)$, $u_3 = a(X_1 + 4X_2 + 2X_3)$ where a is a small quantity. Find the rotation vector.
- d) Show that the principal directions of strain at each point of a linearly elastic isotropic body are coincident with the principal directions of stress.
- e) If u_i is a vector, then show that $\frac{\partial u_i}{\partial x_j}$ is a tensor of second order.
- f) Show that for the velocity field given by $v_1 = ax_3 - bx_2$, $v_2 = bx_1 - cx_3$, $v_3 = cx_2 - ax_1$, the motion is rotational. Also find the vortex lines, where a, b, c are constants.
- g) Find out the differential equation of the path line at time t in a fluid medium.

2. For a relativistic particle, find out the components of force perpendicular and parallel to the direction of motion. Also show that in the relativistic region, the acceleration vector is not parallel to the force vector. 10
3. a) Show that the three principal directions of strain are mutually perpendicular, if the corresponding three principal strains are distinct. 5
- b) The stress tensor at a point P is given by
- $$(T_{ij}) = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}.$$
- Determine the principal stresses and principal directions. 5
4. a) Show that a proper time interval is dilated or expanded when measured from some inertial frame other than the proper frame itself. 5
- b) For homogeneous incompressible fluid moving steadily under the action of gravity only, find the Bernoulli's equation along stream line. 5
5. Deduce the equations of equilibrium of a continuum. Also show that the stress tensor is symmetric. 10
6. a) Find the constitutive equations for isotropic homogeneous linearly viscous compressible fluid. 5
- b) Find for viscous fluid the rate of circulation Γ round a closed circuit in a flow with velocity $\vec{v} = (v_1, v_2, v_3)$. 5
7. a) What is stream function? Find the stream function for a two-dimensional source given by the velocity potential $\phi = -\frac{m}{2r} \log_e r$, where ' r ' is distance from the source point. What does ' m ' represent here? 1 + 5 + 1
- b) For the velocity field $v_1 = \frac{ax_1}{1+t}$, $v_2 = \frac{2ax_2}{1+t}$, $v_3 = \frac{3ax_3}{1+t}$, determine the path lines. 3