QP Code: 24/PT/13/VIA

POST-GRADUATE DEGREE PROGRAMME

Term End Examination — December, 2024

MATHEMATICS

Paper-6A : GENERAL TOPOLOGY

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Let (X, τ) be a topological space and Y be a subspace of X. If A is closed in Y and Y is closed in X, then show that A is closed in X.

   b) Consider the subset $A = \left\{ 1 \pm \frac{1}{n} \,\middle|\, n \in \mathbb{N} \right\}$ of real line $\mathbb{R}$ with usual topology. Find the closure of A in $\mathbb{R}$.

   c) Prove that $\overline{A} = A \cup A'$, where $\overline{A}$ denotes the closure of A and A' denotes the derived set of A.

   d) Show that every second countable topological space is separable.

   e) Show that if x is a limit point of a subset A of some topological space, then $A \setminus \{x\}$ is a member of some filter converging to x.

   f) Let $f : (X, \tau) \to (Y, \tau')$ be a continuous function and $\mathcal{B}$ be a compact subset of Y. Verify whether $f^{-1}(\mathcal{B})$ is always compact or not.

   g) Examine whether the set of reals with co-finite topology is connected or not.

2. a) Let (X, τ) be a topological space and $f : (X, \tau) \to Y$ be an onto function. Show that the following collection $\tau_Y = \left\{ U \subset Y \mid f^{-1}(U) \in \tau \right\}$ of subsets of Y forms a topology on Y such that $f : (X, \tau) \to Y$ is continuous. 4

   b) Show that if every net in a topological space (X, τ) converges to atmost one point in X, then (X, τ) is T_2. 4

   c) Show that the continuous image of a compact set is compact. 2

3. a) Prove that a topological space (X, τ) is a Tychonoff space if and only if it is homeomorphic to a sub-space of a cube. 3

   b) Show that every closed set in a compact set is compact. 4

   c) Show that arbitrary union of compact sets need not be compact. 3

4. a) Prove that a subset of $\mathbb{R}$ with usual topology is connected if and only if it is an interval. 5

   b) Prove that a 1 – 1 continuous function of a compact space onto a T_2-space is a homeomorphism. 3

   c) Show that the subset $\mathbb{Q}$ of all rationals in real number space with usual topology is disconnected. 2

5. a) Show that a space (X, τ) is disconnected iff there is a continuous function $f:(X, \tau) \to \{0,1\}$ which is onto. 3

   b) State and prove Uryshon's lemma. 7

6. a) Prove that components in a topological space must be closed and distinct components should be disjoint. 3

   b) For a subset A of a topological space (X, τ) prove that $(\overline{\overline{A}}) = \overline{A}$. 3

   c) Define product topology. Prove that the product space $(X \times Y, \tau \times \tau')$ is connected if and only if both (X, τ) and (Y, τ') are connected. 4

7. a) Show that every metric space is a uniform space. 4

   b) Show that the real number space $\mathbb{R}$ with usual topology τ is second countable. 6