POST-GRADUATE DEGREE PROGRAMME
Term End Examination — December, 2024
MATHEMATICS
Paper-8A : DIFFERENTIAL GEOMETRY

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

(*Notations and symbols have their usual meanings.*)

Answer Question No. **1** and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Evaluate δ_i^i for $i = 1, 2, 3, 4$.

   b) Prove that if A_i is covariant vector, then $\frac{\partial A_i}{\partial x^j}$ is not a tensor.

   c) Show that the outer product of two tensors A_j^i and B_k^{pq} is a tensor of type (3, 2).

   d) Find the metric of an Euclidean Space referred to cylindrical co-ordinates.

   e) Evaluate :
   $\left(g_{ij}\, g_{kl} - g_{il}\, g_{jk}\right) g^{ij}$

   for a Riemannian space of dimension n.

   f) Define auto-parallel curve.

   g) What do you mean by a developable surface ?

2. a) Define Symmetric and Skew-symmetric tensors. Show that a tensor of order two is expressible as a sum of two tensors one of which is symmetric and the other is antisymmetric. 5

b) Assume that $A(p,q)B_{qj} = C_{pj}$, where B_{qj} is an arbitrary tensor and C_{pj} is a covariant tensor of order two. Show that $A(p, q)$ is a mixed tensor. 5

3. a) Define fundamental metric tensor. Show that the fundamental metric tensor is a symmetric covariant tensor of order (0, 2) 5

b) Prove that the length of a vector is invariant. 5

4. a) State the Serret-Frenet formulii for a space curve. 4

b) Find the curvature at any point of the curve C where

$c: x^1 = a,\ x^2 = t,\ x^3 = bt$ where

$ds^2 = (dx^1)^2 + (x^1)^2\ (dx^2)^2 + (dx^3)^2$, a, b being constant. 6

5. a) Establish first fundamental form of a surface. 5

b) Find the first fundamental form for the surface

$x^1 = a\cos u^1,\ x^2 = a\sin u^1,\ x^3 = u^2$, a being constant. 5

6. a) Determine whether the surface with the metric

$ds^2 = (u^2)^2 (du^1)^2 + (u^1)^2 (du^2)^2$

is developable or not. 6

b) Find the tangent vector for a curve

$c: u^1 = u_0^1, \qquad u^2 = \dfrac{s}{a\cos u_0^1}$. 4

7. a) Show that a surface is a sphere if and only if the second fundamental form is a non-zero constant multiple of its first fundamental form. 5

b) Prove that

$C - 2HB + KA = 0$

where the notations have their usual meanings. 5